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Two-dimensional boson–fermion mixtures in harmonic traps

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Abstract

The density profiles of bosonic and fermionic components in a system of trapped two-dimensional (2D) boson–fermion (BF) mixture are studied. We employ the variational approach to minimize the total energy functional of the BF mixture subject to the conservation of particle numbers of the species. We consider repulsive interactions between bosons and investigate the repulsive and attractive interactions between bosons and fermions. Our results are qualitatively similar to those in 3D, despite the fact that the structure of equations in 2D are different.

Keywords: Bose–Einstein condensation; Boson–fermion mixtures; Phase separation

The study of trapped boson–fermion (BF) mixtures is gaining attention in recent years because of the advances in sympathetic cooling techniques of the fermionic isotopes of atomic gases [1]. As in the binary mixtures of Bose condensates various combinations of interaction strengths among the species in BF mixtures offer the possibility of a rich phase diagram. Such trapped gases are also expected to provide information on the interplay between statistics and interaction effects. Interesting possibilities related to phase separation and temperature effects were put forward for BF mixtures [2]. Experimental efforts have culminated in producing BF mixtures using Li isotopes [3] and recently Na–Li mixtures [4]. Theoretical calculations reported density profiles of the bosonic and fermionic components of the mixture, finite temperature effects and various instabilities for these systems [5–8].

Previous theoretical calculations considered three-dimensional (3D) systems. Varying the trapping field so that it is very narrow in one direction, one may separate the single-particle states of the oscillator potential into well-defined bands, and occupying the lowest band should produce an effectively two-dimensional (2D) system. Recent experiments [9] point to the possibility of realizing 2D trapped atomic gases, and explorations of BF mixtures in similar structures are expected to follow.

The purpose of this paper is to study the ground-state static properties of a 2D trapped BF mixture. We consider a mixture under a common harmonic trap potential at $T=0$. The bosons are in the Bose–Einstein condensed state and fermions are fully spin polarized. Under these circumstances we may neglect the interactions among the fermions. We further assume that particles of both species have the same mass m and introduce the length unit $a_{\mathrm{HO}}=(\hbar/m\omega)^{1/2}$ and energy unit $\hbar\omega$ for scaling purposes. In these units we can write down the total energy functional for the mixture as

$$E = 2\pi \int \mathrm{d}r\, r \left\{ \frac{1}{2}|\nabla\psi|^2 + \frac{1}{2}r^2|\psi|^2 + \frac{1}{2}g|\psi|^4 + \frac{1}{16\pi}(4\pi n_{\mathrm{F}})^2 + \frac{1}{2}r^2 n_{\mathrm{F}} + h n_{\mathrm{F}}|\psi|^2 \right\}, \tag{1}$$

where g and h describe the boson–boson and boson–fermion interactions, respectively. In this equation we have used the Thomas–Fermi approximation for fermions. To develop a variational calculation of the density profiles in a BF mixture, we assume that the condensate wave function is Gaussian, $\psi=(N_{\mathrm{B}}2\alpha/\pi)^{1/2}\mathrm{e}^{-\alpha r^2}$, with α the variational parameter. This should be a reasonable assumption when the number of bosons N_{B} is not too large. Because we treat the fermion density distribution within the TF approximation, there is a cutoff distance R beyond which the fermion density

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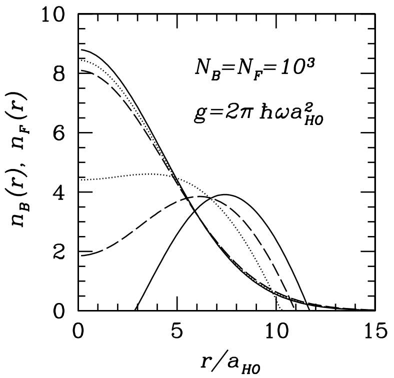

Fig. 1. The density distribution of bosons (Gaussian line shapes) and fermions for the BF interaction strength $h = 0.5\hbar\omega a_{\mathrm{HO}}^2$ (dotted), $h = \hbar\omega a_{\mathrm{HO}}^2$ (dashed), and $h = 1.5\hbar\omega a_{\mathrm{HO}}^2$ (solid).

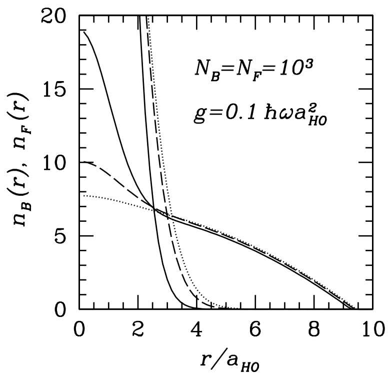

Fig. 2. The density distribution of bosons (only the tails are shown) and fermions for the BF interaction strength $h = -0.5\hbar\omega a_{\mathrm{HO}}^2$ (dotted), $h = -2\hbar\omega a_{\mathrm{HO}}^2$ (dashed), and $h = -5\hbar\omega a_{\mathrm{HO}}^2$ (solid).

vanishes. It is determined implicitly by the equation

$$\varepsilon_{\mathrm{F}} - \frac{1}{2}R^2 - hN_{\mathrm{B}}\frac{2\alpha}{\pi}\mathrm{e}^{-2\alpha R^2} = 0. \tag{2}$$

Finally, the total number of fermions N_{F} is given by

$$N_{\mathrm{F}} = \int_0^R \mathrm{d}r\, r\left(\varepsilon_{\mathrm{F}} - \frac{1}{2}r^2 - hN_{\mathrm{B}}\frac{2\alpha}{\pi}\mathrm{e}^{-2\alpha r^2}\right) \tag{3}$$

where ε_{F} is the Fermi energy.

We have minimized the above total energy functional with respect to the variational parameter α using the number of particles for the species as constraints. In this work we specialize in the repulsive boson–boson interaction ($g > 0$) and consider the repulsive ($h > 0$) and attractive ($h < 0$) cases for the BF interaction. Fig. 1 shows the density distributions of bosonic and fermionic components in a mixture with $N_{\mathrm{B}} = N_{\mathrm{F}} = 10^3$ and $g = 2\pi\hbar\omega a_{\mathrm{HO}}^2$. As the interaction strength between the bosons and fermions is increased we find that the fermionic component is expelled from the center. Because a Gaussian ansatz is used for the boson density we do not readily observe a complete phase separation. In Fig. 2 we consider attractive interactions between bosons and fermions. With increasing $|h|$, we find that the fermion density develops a peak at the origin. The behavior we observe in Figs. 1 and 2 for 2D BF mixtures is qualitatively the same as in 3D systems [8,9]. We note that the dependence of the total energy functional (Eq. (1)) on $n_{\mathrm{F}}(r)$ is quite different than its counterpart in 3D. In conclusion, the basic properties of BF mixtures are preserved in 2D. Further detailed work will be useful for forthcoming experiments on 2D mixtures.

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